Cognitive Diagnostic Assessment - Informing Responses and Interventions

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Agenda

- Some issues with CBM
- Statistic Methods and Models
- Cognitive diagnostic assessment
- Cognitive models
- Example
Curriculum Based Measurement

- Based on fluency
- Standardized
- Drawn from student’s curriculum
- Sensitive to change
- Not intended to be diagnostic
Changes in CBM

- Development of local norms
- Identification of benchmarks
- Development of general probes
- Use in program evaluation
- Use in response to intervention models
- Use in special education eligibility decisions
Problems with CBM Slope

Ardoin & Christ (2009):
- Research is on groups, not individuals
- Confidence intervals for individual data are wider than data variability

Lembke, Foegen, Whittaker, & Hampton (2008):
- Slopes did not differ between students
- Slopes were not necessarily linear

Yeo, Fearrington, & Christ (2011):
- Slopes from two types of reading probes were uncorrelated
- Slopes were unstable over time within measures
CBM Data Collection

Monaghen, Christ, & Van Norman (2012):
- Little data on decision rules for CBM; recommendations are overly optimistic
- Data are hard to collect frequently
- Instructional effects take time to manifest
- 2 to 5 x weekly for 8 weeks or more
Scores v. Growth

Tran, Sanchez, Arellano, & Swanson (2011):

- Pretest scores predicted posttest scores regardless of intervention
- Achievement gap was maintained between low responders and adequate responders
- RTI intervention and progress monitoring did not improve prediction of low response over pretest scores
Unidimensionality of Probes

Christ, Scullin, Tolbize, & Jiban (2008):
- Most math probes assess subskill mastery rather than general outcomes
- Not yet known whether CBM math can predict math proficiency as reading fluency probes predict overall reading proficiency

Foegen, Jiban, & Deno (2007):
- Most CBM math is curriculum sampling useful for tracking individual skill development
- Robust indicators will be necessary for predicting broad math outcomes
More research needed on CBM math
All measurement contains error; CBM contains large amounts
CBM math probes usually unidimensional; correspondence to broad outcomes unknown
CBM data are unstable when used to show growth for individual students
CBM is not diagnostic
CBM does not tell us what kids don’t know
Scientific Thinking

- What is it that we want to know?
- What evidence will address our questions?
- Collecting data is not enough.
Statistical Methods and Models

- Dimensionality

DiBello, 2007
Dimensionality

- Unidimensional theories assume a single underlying ability or latent trait that determines test responses.

- Multidimensional theories assume multiple underlying abilities or latent traits that work in combination to determine test responses.

- Is mathematics unidimensional or multidimensional?
Statistical Methods and Models

- Dimensionality
- Q-matrix
## Q Matrix Example

<table>
<thead>
<tr>
<th>Item #</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
<th>A5</th>
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</table>
Statistical Methods and Models

- Dimensionality
- Q-matrix
- Assumptions
  - Conjunctive
  - Disjunctive
Conjunctivity

Conjunctive

- Correct responses are assumed to occur when all “required” attributes are mastered

Disjunctive

- Correct responses may occur when one or more “required” attributes are mastered
Statistical Methods and Models

- Dimensionality
- Q-matrix
- Assumptions that
  - Conjunctive
  - Disjunctive
  - Compensatory
  - Noncompensatory
Compensation

Noncompensatory

- Ability on one attribute **does not** make up for lack of ability on other attributes.

Compensatory

- Ability on one or more attributes **can** make up for lack of ability on other attributes.
Statistical Methods and Models

- Dimensionality
- Q-matrix
- Assumptions that
  - Conjunctive
  - Disjunctive
  - Compensatory
  - Noncompensatory
  - Slipping
  - Guessing
Slipping and Guessing

- Slips = errors
  - Each cognitive diagnostic model (CDM) contains a parameter that estimates the likelihood that a student simply made a mistake when answering an item.

- Guessing
  - Most CDMs contain a parameter that estimates the likelihood that a student guessed the correct answer to an item.
Cognitive Diagnostic Assessment

Psychology of Learning

Statistical methods and models

Inferences about students' specific knowledge structures and processing skills

Alves, 2012
Steps in the Process

1. Develop a **cognitive model**.
2. Construct test items that are sensitive to the cognitive model.
3. Administer test items.
4. Analyze responses to
   - Evaluate the plausibility of the model
   - Describe students’ knowledge according to strengths and weaknesses
Cognitive Models

- Theoretical maps of how people learn and organize content knowledge.

- **New things** are learned most easily when they can be **connected to existing knowledge**.

- Cognitive models are useful tools for guiding instruction and assessment.
Types of Cognitive Models

- **Linear models**
  - Learning progressions (Popham, 2008, 2011; Wilson, 2009)
  - Construct maps (Wilson, 2009)

- **Network models**
  - Attribute hierarchies (Leighton, Gierl, & Hunka, 2004)
  - Learning hierarchies (Gagné, 1968)
  - Learning maps (dynamiclearningmaps.org, 2010)
Learning Progression

- Enabling knowledge
- Subskill(s)
- Target Curricular Aim
Construct Map

Most Proficiency

Level 4
Level 3
Level 2
Level 1

Least proficiency
Learning Hierarchy
Consider Grain Size

- Cognitive models can be developed using different levels of detail or grain sizes.
- Different grain sizes may be appropriate for different purposes:
  - Describing a person’s cognition
  - Instructional planning
  - Assessment development
  - Interpreting assessment observations/test responses
Three Phases for Mastering Basic Number Computations

(Baroody, 2006)

1.0A.5 - Relate counting to addition and subtraction

Counting strategies

Using object counting or verbal counting to determine an answer

Reasoning strategies

Using known information to logically determine the answer of an unknown situation.

Mastery

Efficient (fast and accurate) production of answers

2.0A.1 - Use addition and subtraction within 100 to solve one- and two-step word problems...

1.0A.6 - Add and subtract within 20, demonstrating fluency for addition and subtraction within 10...

2.NBT.5 - Fluently add and subtract within 100 using strategies...
Counting strategies

Using object counting or verbal counting to determine an answer

Reasoning strategies

Using known information to logically determine the answer of an unknown situation.

Mastery

Efficient (fast and accurate) production of answers

Baroody, 2006
What do you think?

- What grain size models are appropriate for tools used within the RtI process?
  - Assessment tools
  - Intervention goals
The Assessment Triangle (NRC, 2001)

Observation

Cognition

Interpretation
A logical combination...

Assessment

Feedback

Cognitive Model

Teaching and Learning
Foundational Concepts Related to Slope: An Application of the AHM

- An implementation of the process articulated in the evidence-centered design literature.
- An example of using mathematics education literature to design an cognitive model (e.g., attribute hierarchy).
- An example of test development focused on conceptual knowledge.
- An application of the AHM to actual student test responses.
A concept is a cognitive representation of something that is real (Ausubel, 1968; Bruner, Goodnow & Austin, 1956; Martorella, 1972).

Conceptions mature over time and experience (Martorella, 1972).

Concepts are classified in a variety of ways (Bruner, et al.1956; Henderson, 1970).

Concept learning is influenced by prior knowledge, thinking, and experience (Bruner, et al.,1956; Gagné, 1971; Inhelder & Piaget, 1964).

Misconceptions arise when flawed information or erroneous connections are associated with a concept (Glaser, 1986; Henderson, 1970).

Misconceptions may also be viewed as immature (Klausmeier, 1992; Wilson, 2009).
Slope is Essential Mathematics

- Necessary to work with linear functions
  (National Mathematics Advisory Panel, 2008; NCTM, 2009)

- Necessary for calculus and statistics
  (Wilhelm & Confrey, 2003)

- “One of the most important mathematical concepts students encounter”
  (Joram & Oleson, 2007)
Foundations for Understanding Slope

- **Covariational Reasoning** (Adamson, 2005)
  - Detecting which quantities are related in a mathematical situation
  - Detecting the direction of the relationship in a variation problem

- **Proportional Reasoning** (Kurtz & Karplus, 1979)
  - More than determining a missing number
  - Detecting the constant rate that governs a proportional relationship and using the rate to reason about the quantities in the proportion
Sources of Misconceptions

- Additive reasoning (Heller, Post, Behr, & Lesh, 1990)
- Incorrect quantities identified for the slope ratio (Moritz, 2005)
- Opposite slope (Barr, 1980)
- Reciprocal slope (Barr, 1980)
- Total amount confused with amount of change (Bell & Janvier, 1981)
- Univariate reasoning (Moritz, 2005)
Foundational Concepts of Slope Attribute Hierarchy (FCSAH)

A1: Identify covariates in a problem scenario.
A2: Identify covariates and the direction of their relationship.
A3: Interpret a slope whose value equals a whole number.
A4: Interpret a slope whose value simplifies to a positive unit fraction.
A5: Interpret a slope whose value simplifies to a positive rational number that is neither a whole number nor a unit fraction.
Foundational Concepts of Slope Assessment (FCSA)

<table>
<thead>
<tr>
<th>Item #</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
<th>A5</th>
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</table>
Jill deposits the same amount of money into her savings account every time she goes to the bank. She does not withdraw any money. Which fact about Jill’s trips to the bank is related to the total amount of money she has in her account?

A. the time of day
B. the day of the week
C. the number of deposits
D. the distance to the bank
The graph below shows the speeds and times of students who ran a 2-mile race. Based on the graph, which statement must be true?

A. A student who runs faster uses more time.
B. A student who runs slower uses more time.
C. A student who uses more time runs farther.
D. A student who uses less time runs farther.
The graph below shows the amount of money, in dollars, a class could raise by selling cookie dough.

Based on this graph, which statement must be true?

A. For every 1 bucket sold, the class earns $1.
B. For every 5 buckets sold, the class earns $1.
C. The class earns $1 per bucket of cookie dough.
D. The class earns $5 per bucket of cookie dough.
Sample Characteristics

- 1629 students
  - Pre-algebra - 630 students
  - Algebra 1 - 492
  - Geometry - 365
  - Algebra 2 - 142
- 26 different Kansas school districts
- 30 different teachers
Data Analysis

- Item Response Theory (IRT) - 3 PL
- Attribute Hierarchy Method (AHM) (Leighton, Gierl, & Hunka, 2004)
  - Estimated abilities for 10 expected response patterns consistent with the FCSAH
  - Classified each student into one of the 10 knowledge states consistent with the FCSAH
### Expected Response Vectors

<table>
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<tr>
<th>Knowledge State</th>
<th>Expected Response Vector</th>
<th>Ability Estimate</th>
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Example of the AHM Comparison
(Observed Vector: 11111111111101111110, Ability Estimate = 0.64)

<table>
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<tr>
<th>Ability Estimate</th>
<th>Expected Response Vector</th>
<th>$L_{j\text{Expected}}(\Theta)$</th>
<th>$P_{j\text{Expected}}(\Theta)$</th>
<th>Knowledge State</th>
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<td>A12345</td>
</tr>
</tbody>
</table>
Knowledge State Classifications

![Knowledge State Classifications Diagram]

- Ability (θ)
- Percent Frequency

- A0
- A1
- A12
- A13
- A124
- A125
- A1234
- A1235
- A1245
- A12345
We started with a hierarchy...
...and identified a progression

- Identify two quantities that vary together.
- Determine the direction of the relationship.
- Interpret a unit rate depicted in a graph.
Recommendations

- Mathematics education research should be consulted into the development of theories and cognitive models used in assessment development.

- Instructional planning, responses, and interventions should be sensitive to theories of how students learn.

- Classroom assessments should be developed using the same theories about learning that guide instruction.
Cognitive Models and Curriculum

- Should the cognitive model and assessment tools be associated directly with curriculum materials?
- Is it possible to develop cognitive models to guide instruction that are curriculum agnostic?
Grade Level Considerations

- How far off grade level should assessments go in order to query prerequisite skills and understandings?
Professional Development

- What professional development opportunities in what modalities should be developed for teachers to:
  - Acquaint them with models of how students learn mathematics?
  - Help them plan instruction that is sensitive to how students learn?


References


References